FORMULATION

We start by writing the curl of the electric field.

$$\vec{\nabla} \times \vec{e} = -\mu_o \frac{\partial}{\partial t} \vec{h} \tag{4.2}$$

A physical magnetic device must also satisfy $\vec{\nabla} \cdot \vec{B} = 0,\quad \vec{\nabla} \times \vec{B} = 0$ and therefore $B_{wox},\ B_{woy},\ k_w,\ \phi_{wx}$ and ϕ_{wy} cannot be chosen arbitrarily.

$$\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & -jk_o \\ e_x & e_y & e_z \end{bmatrix} = -\mu_o \frac{\partial}{\partial t} \vec{h} \tag{4.3}$$

The matrix form of the curl equation can be written as three separate equations.

$$\hat{x}\, e_y(-\,(-jk_o)) = \hat{x}\left(-\mu_o \frac{\partial}{\partial t} h_x\right) \tag{4.4a}$$

$$\hat{y}\, e_x(-jk_o) = \hat{y}\left(-\mu_o \frac{\partial}{\partial t} h_y\right) \tag{4.4b}$$

$$\hat{z}\,(0) = \hat{z}\left(-\mu_o \frac{\partial}{\partial t} h_z\right) \tag{4.4c}$$

Substituting $k_o = \omega_o\sqrt{\mu_o\varepsilon_o}$ and $\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}$ and rearranging the terms we can write the equations for the individual components of the magnetic field.

$$\frac{d}{dt}(\eta_o h_x) = -j\omega_o e_y \tag{4.5a}$$

$$\frac{d}{dt}(\eta_o h_y) = j\omega_o e_x \tag{4.5b}$$

$$\frac{d}{dt}(\eta_o h_z) = 0 \tag{4.5c}$$

The curl equation for the magnetic field can also be written in matrix form.

$$\vec{\nabla} \times \vec{h} = \varepsilon_o \frac{\partial}{\partial t}\vec{e} - N_o q \vec{\mathrm{v}} \tag{4.6}$$

$$\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & -jk_o \\ h_x & h_y & h_z \end{bmatrix} = \varepsilon_o \frac{\partial}{\partial t}\vec{e} - N_o q \vec{\mathrm{v}} \tag{4.7}$$

Equation (4.8) can be written as three separate equations.

$$\hat{x} h_y(-(-jk_o)) = \hat{x}\left(\varepsilon_o \frac{\partial}{\partial t} e_x - N_o q \mathrm{v}_x\right) \tag{4.8a}$$

$$\hat{y} h_x(-jk_o) = \hat{y}\left(\varepsilon_o \frac{\partial}{\partial t} e_y - N_o q \mathrm{v}_y\right) \tag{4.8b}$$

$$\hat{z}(0) = \hat{z}\left(\varepsilon_o \frac{\partial}{\partial t} e_z - N_o q \mathrm{v}_z\right) \tag{4.8c}$$

After substituting and rearranging as we did before we can write the equations for the individual components of the electric field.

$$\frac{d}{dt}(e_x) = j\omega_o(\eta_o h_y) + \frac{N_o q}{\varepsilon_o} \mathrm{v}_x \tag{4.9a}$$

$$\frac{d}{dt}(e_y) = -j\omega_o(\eta_o h_x) + \frac{N_o q}{\varepsilon_o} \mathrm{v}_y \tag{4.9b}$$

$$\frac{d}{dt}(e_z) = \frac{N_o q}{\varepsilon_o} \mathrm{v}_z \tag{4.9c}$$

Finally the velocity field equation is given as:

$$\frac{\partial}{\partial t}\vec{\mathrm{v}} = -\frac{q}{m}\vec{e} - \frac{q}{m}\left(\vec{\mathrm{v}} \times \vec{B_w}\right) \tag{4.10}$$

Writing the velocity equation in matrix form we have:

$$\frac{\partial}{\partial t}\vec{\mathrm{v}} = -\frac{q}{m}\vec{e} - \frac{q}{m}\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \mathrm{v}_x & \mathrm{v}_y & \mathrm{v}_z \\ B_{wox}\cos(k_w z + \phi_{wx}) & B_{woy}\sin(k_w z + \phi_{wy}) & 0 \end{bmatrix} \tag{4.11}$$

The matrix equation is written as three separate equations to show the individual velocity components.

$$\frac{\partial}{\partial t}\mathrm{v}_x = -\frac{q}{m}e_x - \frac{q}{m}(-\mathrm{v}_z B_{woy}\sin(k_w z + \phi_{wy}) \tag{4.12a}$$

$$\frac{\partial}{\partial t}\mathrm{v}_y = -\frac{q}{m}e_y - \frac{q}{m}(\mathrm{v}_z B_{wox}\cos(k_w z + \phi_{wx}) \tag{4.12b}$$

$$\frac{\partial}{\partial t}\mathrm{v}_x = -\frac{q}{m}e_x - \frac{q}{m}(\mathrm{v}_x B_{woy}\sin(k_w z + \phi_{wy}) - \mathrm{v}_y B_{wox}\cos(k_w z + \phi_{wx}))$$

(4.12c)

Now we make following substitutions as in section 2.3

$$\omega_{bx} = \omega_{bwx}\cos(k_w z + \phi_{wx}) \quad (4.13)$$

$$\omega_{by} = \omega_{bwy}\sin(k_w z + \phi_{wy}) \quad (4.14)$$

where

$$\omega_{bwx} = \frac{q}{m}B_{wox} \quad (4.15)$$

$$\omega_{bwy} = \frac{q}{m}B_{woy} \quad (4.16)$$

which gives

$$\frac{d}{dt}(\mathrm{v}_x) = -\frac{q}{m}e_x + \omega_{by}\mathrm{v}_z \quad (4.17a)$$

$$\frac{d}{dt}(\mathrm{v}_y) = -\frac{q}{m}e_y - \omega_{bx}\mathrm{v}_z \quad (4.17b)$$

$$\frac{d}{dt}(\mathrm{v}_z) = -\frac{q}{m}e_z - \omega_{by}\mathrm{v}_x + \omega_{bx}\mathrm{v}_y \quad (4.17c)$$

Equations (4.5b), (4.9a), (4.9c) and (4.15a) are the same as (2.7a) through (2.7d). Equation (4.15c) differs from (2.7e) only by the $\omega_{bx}\mathrm{v}_y$ term. This system of equations is taken as a perturbation and gives us the extraordinary waves.

The set of equations (4.5a), (4.9b) and (4.15b) define a system of equations involving h_x, e_y, v_y and v_z. These equations give us the ordinary waves and again there is also the ω_{bx} coupling term. Note that if ω_{bx} is zero we have the same situation as in chapter two and there is no coupling. The matrix equations of (4.3), (4.7) and (4.11) can be combined into a single matrix

$$\frac{d}{dt}\begin{bmatrix} \eta_o h_x \\ \eta_o h_y \\ \eta_o h_z \\ e_x \\ e_y \\ e_z \\ v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -j\omega_o & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & j\omega_o & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & j\omega_o & 0 & 0 & 0 & 0 & \frac{N_o q}{\varepsilon_o} & 0 & 0 \\ -j\omega_o & 0 & 0 & 0 & 0 & 0 & 0 & \frac{N_o q}{\varepsilon_o} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{N_o q}{\varepsilon_o} \\ 0 & 0 & 0 & \frac{-q}{m} & 0 & 0 & 0 & 0 & -\omega_{by} \\ 0 & 0 & 0 & 0 & \frac{-q}{m} & 0 & 0 & 0 & \omega_{bx} \\ 0 & 0 & 0 & 0 & 0 & \frac{-q}{m} & -\omega_{by} & \omega_{bx} & 0 \end{bmatrix} \begin{bmatrix} \eta_o h_x \\ \eta_o h_y \\ \eta_o h_z \\ e_x \\ e_y \\ e_z \\ v_x \\ v_y \\ v_z \end{bmatrix} \qquad (4.18)$$

The 9 x 9 matrix can be reduced to an 8 x 8 matrix by removing the row corresponding to equation (4.5c) since these values are all zero. The matrix can also be rearranged to obtain better insight into the coupling terms.

$$\frac{d}{dt}\begin{bmatrix} e_x \\ \eta_o h_y \\ e_z \\ v_x \\ v_z \\ v_y \\ e_y \\ \eta_o h_x \end{bmatrix} = \left[\begin{array}{ccccc|ccc} 0 & j\omega_o & 0 & \frac{N_o q}{\varepsilon_o} & 0 & 0 & 0 & 0 \\ j\omega_o & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{N_o q}{\varepsilon_o} & 0 & 0 & 0 \\ -\frac{q}{m} & 0 & 0 & 0 & \omega_{by} & 0 & 0 & 0 \\ 0 & 0 & -\frac{q}{m} & -\omega_{by} & 0 & \omega_{bx} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & -\omega_{bx} & 0 & -\frac{q}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{N_o q}{\varepsilon_o} & 0 & -j\omega_o \\ 0 & 0 & 0 & 0 & 0 & 0 & -j\omega_o & 0 \end{array}\right] \begin{bmatrix} e_x \\ \eta_o h_y \\ e_z \\ v_x \\ v_z \\ v_y \\ e_y \\ \eta_o h_x \end{bmatrix} \qquad (4.19)$$

In chapter 2, the solution was split into two problems, where one problem corresponded to x component of the incident wave, and the other problem corresponded to the y component of the incident wave. The individual solutions were then combined by using superposition. We can use the same method in this case, provided that the x component of the magnetic wiggler field is small compared to the y component. We can state this mathematically by letting $\omega_{bx} \ll \omega_{by}$. By setting this limitation, the coupling term ω_{bx} in 4.19 can be neglected and there is an uncoupling of the two systems allowing us to solve the systems independently however the input to system 2 when system 1 alone is excited is influenced by the coupling. A similar remark applies when system 2 alone is excited. The solution of this problem will be pursued soon.