3.2 FORMULATION AND SOLUTION

The effect of switching on a *lossy* plasma medium on waves and wiggler fields may be examined by introducing a collision frequency ν, in the plasma field equations. The momentum equations (2-7d) and (2-7e) get modified as follows:

$$\frac{d}{dt}(v_x) = -\frac{q}{m}e_x + \omega_b v_z - \nu v_x \tag{3.1a}$$

$$\frac{d}{dt}(v_z) = -\frac{q}{m}e_z - \omega_b v_x - \nu v_z \tag{3.1b}$$

In the absence of collisions ($\nu = 0$), the real frequencies of the newly created waves can be obtained by replacing $\left(\frac{d}{dt}\right)$ by $j\omega$ in (2-7). This yields the equation given in (2-12). When collisions are included, each of the frequencies will be associated with a damping term, thus forming a complex frequency, $s = \sigma + j\omega$ [9]. Therefore in order to obtain the complex frequencies, it is necessary to replace $\left(\frac{d}{dt}\right)$ by s in (2-7a),(2-7b),(2-7c),(2-25a) and (2-25b). The characteristic equation for the complex frequency variable satisfying the above equation is obtained as:

$$D(s) = s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \tag{3.2a}$$

where

$$a_4 = 2\nu \tag{3.2b}$$

$$a_3 = (\omega_o^2 + \nu^2 + \omega_b^2 + 2\omega_p^2) \tag{3.2c}$$

$$a_2 = 2\nu(\omega_o^2 + \omega_p^2) \tag{3.2d}$$

$$a_1 = \omega_o^2(\nu^2 + \omega_b^2 + \omega_p^2) + \omega_p^4 \tag{3.2e}$$

$$a_o = \nu\omega_o^2\omega_p^2 \tag{3.2f}$$

The roots of D(s) are all complex. The imaginary parts of the roots of D(s) correspond to harmonic variation with time while their real parts indicate exponential decay with time. It is possible to obtain the roots of D(s) analytically, but the resulting expressions are expected to be mathematically complicated. In order to facilitate ease in the computation of the roots with the consequent advantage of ease in recognizing the effect of losses on the shift in the roots, the collisions frequency, ν , is restricted to take small values. This restriction implies the imposition of a low loss magnetoplasma medium. Taylor series expansion of s about $\nu = 0$ gives the relation:

$$s(\nu) = [s]_{\nu=0} + \nu\left\{\left[\frac{ds}{d\nu}\right]_{\nu=0}\right\} \tag{3.3}$$

The first term in the right hand side of (3-3) corresponds to the frequencies of the four waves given in (2-12) and the second term gives the associated damping factor. The four complex roots of (3-2a) are therefore given by:

$$s_n = \sigma_n + j\omega_n \qquad n = 1,2,3,4 \qquad (3.4a)$$

$$\sigma_n = \nu\left\{\left[\frac{ds}{d\nu}\right]_{\nu=0,s_n=j\omega_n}\right\} \qquad n = 1,2,3,4 \qquad (3.4b)$$

Differentiation of the polynomial (3-2a) with respect to ν gives the expression for σ_n [8]:

$$\sigma_n = \nu\left[\frac{2\omega_n^2(\omega_n^2-\omega_o^2-\omega_p^2)+\omega_o^2\omega_p^2}{5\omega_n^4-3\omega_n^2(\omega_o^2+\omega_b^2+2\omega_p^2)+(\omega_p^4+\omega_o^2(\omega_b^2+\omega_p^2))}\right] \qquad n = 1,2,3,4 \qquad (3.5)$$

The decay rates of the third wave and the fourth wave are the same as those of the first wave and the second wave respectively since $\omega_1 = -\omega_3$ and $\omega_2 = -\omega_4$. An approximate expression for the imaginary part of the fifth root of (3-2a) is obtained by substituting $\nu = 0$ in (3-2a). Thus we get $\omega_5 = 0$. The associated fields are the wiggler fields. The decay rate of the wiggler field is obtained by substituting $\omega_n = 0$ in (3-5). Therefore the y component of the magnetic wiggler field and the electric wiggler field damp out as:

$$\exp(-\sigma_{wX}t) \qquad (3.6a)$$

where

$$\sigma_{wX} = \frac{\nu\omega_o^2\omega_p^2}{\omega_p^4+\omega_o^2(\omega_b^2+\omega_p^2)} \qquad (3.6b)$$

The damping rate of the x component of the wiggler magnetic field is obtained by substituting $\omega_b = 0$ in (3-6b). The x component of the magnetic wiggler field damps out as:

$$\exp(-\sigma_{wO}t) \tag{3.7a}$$

where

$$\sigma_{wO} = \frac{\nu\omega_o^2}{\omega_p^2 + \omega_o^2} \tag{3.7b}$$

Note that the subscripts X and O are used to designate the wiggler fields associated with the extraordinary waves and the ordinary waves respectively.