2.3 SOLUTION

The problem may be solved using superposition by splitting the incident wave into separate components. Problem 1 will be the solution when the input wave has the x component of the electric field and Problem 2 will be the solution when the input wave has the y component of the electric field.

<u>Problem 1</u>:

$$\overrightarrow{e_{i1}} = \hat{x} p E_o \exp[j(\omega_o t - k_o z)] \quad t < 0 \tag{2.5}$$

$$\overrightarrow{h_{i1}} = \hat{y} p H_o \exp[j(\omega_o t - k_o z)] \quad t < 0 \tag{2.6}$$

$$\overrightarrow{B_w} = \hat{y} B_{wo} \sin(k_w z + \phi_w) \tag{2.7}$$

At t = 0, a plasma of frequency ω_p is switched on. The problem may be solved along the lines of the solution given in [8]. Expressing the fields in the form of:

$$f(z,t) = f(t)\exp(-jk_o z) \tag{2.8}$$

we can write the plasma fields for t > 0 in differential form:

$$\frac{d}{dt}(\eta_o h_y) = j\omega_o e_x \tag{2.9a}$$

$$\frac{d}{dt}(e_x) = j\omega_o(\eta_o h_y) + \frac{N_o q}{\varepsilon_o}\mathrm{v}_x \tag{2.9b}$$

$$\frac{d}{dt}(e_z) = \frac{N_o q}{\varepsilon_o}\mathrm{v}_z \tag{2.9c}$$

$$\frac{d}{dt}(\mathrm{v}_x) = -\frac{q}{m}e_x + \omega_b \mathrm{v}_z \tag{2.9d}$$

$$\frac{d}{dt}(\mathrm{v}_z) = -\frac{q}{m}e_z - \omega_b \mathrm{v}_x \tag{2.9e}$$

where

$$\omega_b = \omega_{bw}\sin(k_w z + \phi_w) \tag{2.10}$$

and

$$\omega_{bw} = \frac{qB_{wo}}{m} \tag{2.11}$$

and $\eta_o = 120\pi$ is the intrinsic impedance of free space.

Solving equation (2.9a-e), using the Laplace Transform technique of [8], shows that four extraordinary waves (X waves) and two wiggler fields are generated. The two wiggler fields in terms of the four new frequencies are given by:

$$\frac{e_{zw}(z,t)}{E_o} = jp\frac{\omega_o\omega_b\omega_p^2}{\omega_1\omega_2\omega_3\omega_4}\exp(-jk_o z) \tag{2.12}$$

$$\frac{h_{yw}(z,t)}{H_o} = p\frac{\omega_p^4}{\omega_1\omega_2\omega_3\omega_4}\exp(-jk_o z) \tag{2.13}$$

where $\omega_1, \omega_2, \omega_3,$ and ω_4 are the roots of the polynomial [8] :

$$\omega^4 - \omega^2(\omega_o^2 + \omega_b^2 + 2\omega_p^2) + [\omega_p^4 + \omega_o^2(\omega_b^2 + \omega_p^2)] = 0 \tag{2.14}$$

<u>Problem 2</u>:

$$\overrightarrow{e_{i2}} = \mp \hat{y} q E_{i2} \exp[j(\omega_o t - k_o z)] \qquad t < 0 \tag{2.15}$$

$$\overrightarrow{B_w} = \hat{y} B_{wo} \sin(k_w z + \phi_w) \tag{2.16}$$

where

$$E_{i2} = jE_o \tag{2.17}$$

$$\overrightarrow{h_{i2}} = \pm \hat{x} q \frac{E_{i2}}{\eta_o} \exp[j(\omega_o t - k_o z)] \tag{2.18}$$

$$= \hat{x} q H_{i2} \exp[j(\omega_o t - k_o z)] \tag{2.19}$$

where

$$H_{i2} = \pm \frac{jE_o}{\eta_o} = \pm jH_o \ . \tag{2.20}$$

At t = 0, a plasma of frequency ω_p is switched on. This problem may be solved as in Problem 1 and using the technique of [9] we obtain two ordinary waves (O waves) and a magnetic wiggler field [9],[11],[12]. The frequencies of the ordinary waves are given by:

$$\omega = \pm \sqrt{\omega_o^2 + \omega_p^2} \tag{2.21}$$

Note that there is no electric wiggler field.

The magnetic wiggler field is:

$$\frac{h_{xw}(z,t)}{H_o} = \frac{\omega_p^2}{\omega_o^2 + \omega_p^2} \frac{H_{i2}}{H_o} \exp(-jk_o z) = \pm jq \frac{\omega_p^2}{\omega_o^2 + \omega_p^2} \exp(-jk_o z) \tag{2.22}$$

The total wiggler fields can be found by superposing the results of Problem 1 and Problem 2. The total magnetic wiggler field is:

$$\frac{\overrightarrow{h_w(z)}}{H_o} = \mathbf{Re}\left[\hat{y}p\frac{\omega_p^4}{\omega_1\omega_2\omega_3\omega_4}\exp(-jk_oz) \mp \hat{x}\,jq\frac{\omega_p^2}{\omega_o^2+\omega_p^2}\exp(-jk_oz)\right] \tag{2.23}$$

$$= \hat{y}p\frac{\omega_p^4}{\omega_1\omega_2\omega_3\omega_4}\cos(k_oz) \pm \hat{x}q\frac{\omega_p^2}{\omega_o^2+\omega_p^2}\sin(k_oz) \tag{2.24}$$

$$= \hat{y}p\frac{\omega_p^4}{\omega_p^4+\omega_o^2\left(\omega_{bw}^2\sin^2(k_wz+\phi_w)+\omega_p^2\right)}\cos(k_oz) \pm \hat{x}q\frac{\omega_p^2}{\omega_o^2+\omega_p^2}\sin(k_oz) \tag{2.25}$$

The total electric wiggler field is:

$$\frac{e_w(z)}{E_o} = p\frac{\omega_o\omega_b\omega_p^2}{\omega_p^4+\omega_o^2\left(\omega_{bw}^2\sin^2(k_wz+\phi_w)+\omega_p^2\right)}\sin(k_oz) \tag{2.26}$$